MATH 2020 Advanced Calculus II

Tutorial 1

1. Compute $\displaystyle\iint_R (x^2+1)y\,dA$, where $R = \{0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant 2\}$.

   Solution.

$$\begin{aligned}
\iint_R (x^2+1)y\,dA &= \int_0^2\int_0^1 (x^2+1)y\,dx\,dy \\
&= \left(\int_0^1 (x^2+1)\,dx\right)\left(\int_0^2 y\,dy\right) \\
&= \frac{8}{3}
\end{aligned}$$

2. Compute $\displaystyle\iint_R ye^{xy}\,dA$, where $R = \{0 \leqslant x, y \leqslant 1\}$.

   Solution. Notice that in the integrand there is the term y which absorbs $\frac{1}{y}$ coming from integrating e^{xy} with respect to x, and so it is more appropriate to compute the double integral by integrating with respect to x first.

$$\begin{aligned}
\iint_R ye^{xy}\,dA &= \int_0^1\int_0^1 ye^{xy}\,dx\,dy \\
&= \int_0^1 \left[e^{xy}\right]_0^1 dy \\
&= \int_0^1 (e^y - 1)\,dy \\
&= e - 2
\end{aligned}$$

3. Find the volume of the solid bounded above by the surface $z = 25 - x^2 - y^2$ and below by $R = \{-3 \leqslant x \leqslant 3,\ -4 \leqslant y \leqslant 4\}$.

   Solution. Notice that the given surface intersects the xy-plane along the circle of radius 5 which bounds a disk containing the region R (in fact, the four vertices of R are $(\pm 3, \pm 4)$ which all lie on this circle).

$$\begin{aligned}
\text{volume} &= \iint_R z\,dA \\
&= \int_{-4}^4\int_{-3}^3 (25 - x^2 - y^2)\,dx\,dy \\
&= \int_{-4}^4 \left[25\times 6 - \frac{3^3\times 2}{3} - 6y^2\right] dy \\
&= (150 - 18)\times 8 - 6\times\frac{4^3\times 2}{3} \\
&= 800
\end{aligned}$$